Nucleon-nucleus elastic scattering at near-GeV energies within a microscopic folding approach

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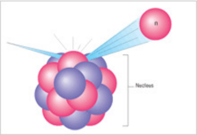
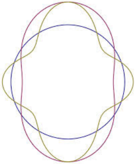
Workshop on Nuclear Reactions (Theory and Experiment)
ENSAR2 - NUSPRASEN, Warsaw, Jan 22-24 2018

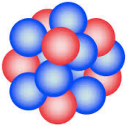
V_{NN} V_{NNN} ...



Collisions and reactions



Excitations



Structure

Astrophysics; neutron stars

Problem, motivation & assumptions

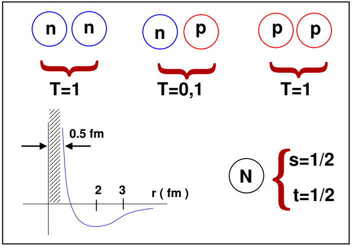
Questions

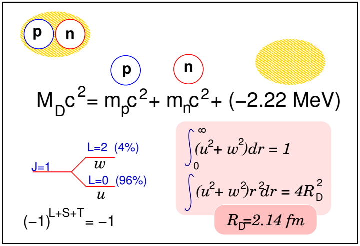
- How reasonable can the microscopic optical model for pA collisions be at energies up to $\sim$1 GeV, taking into account NN phase-shift analyses above pion production threshold? ($E_{Lab}/2 \sim m_\pi = 140$ MeV, $q_0 \approx 2$ fm^{-1})
- Can geometric properties of the g.s. be inferred from such collisions? (densities, radii, ...) FAIR-GSI 0.2-1 A GeV (DPhN-Saclay)
- Can reliable predictions be made for experiments involving radioactive beams colliding hydrogen targets?

Considerations

- Use of minimal relativity within non-relativistic framework for the optical model potential (NN and pA)
- Realistic bare NN interaction (AV18, Paris, NIJ, Bonn, chiral, etc) plus separable contribution
- Lowest order in BHF or impulse approximation
- Retain full nonlocalities of the NN effective interaction and optical potential (no rescaling nor localization)

NN data constraints for *realistic* V_{NN}





- Deuteron, only *NN* bound state in free space: $T = 0$; $S = 1$; $J^\pi = 1^+$.
- Phase-shifts $\delta_L(E)$ for... $0 < E \lesssim 2m_\pi$ $\cdots$ $2m_\pi \lesssim E < \cdots$
- Static properties ^3H, etc.

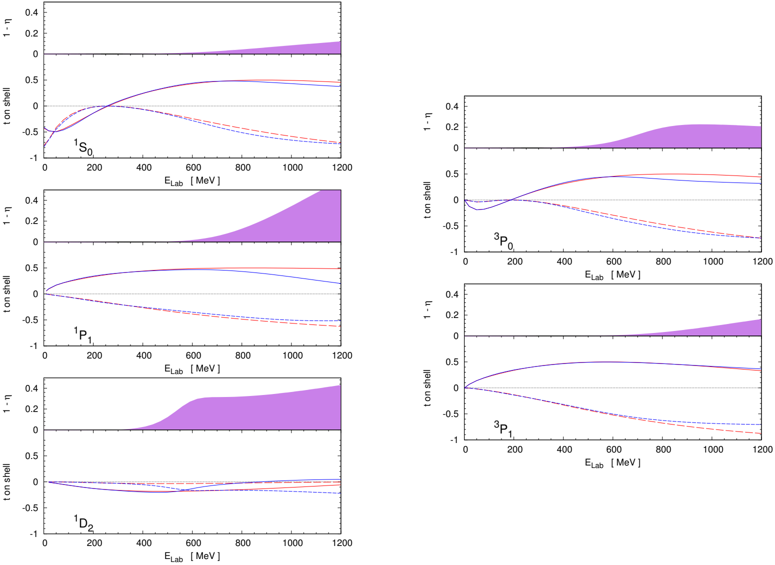
NN/Partial Wave Amplitudes at INS DAC: http://gwdac.phys.gwu.edu

Realistic *NN* potential models

- Paris ($\lesssim 330$ MeV)
- Bonn A and B ($\lesssim 300$ MeV)
- Nijmegen I and II ($\lesssim 350$ MeV)
- Argonne $v18$ ($\lesssim 350$ MeV)
- CD Bonn ($\lesssim 350$ MeV)
- Chiral ($\lesssim 290$ MeV)

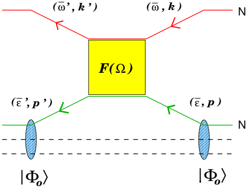
Lacombe *et al.* PRC21, 861 (1980)
Machleidt *et al.* PhysRep149, 1 (1987)
Stoks *et al.*, PRC49, 2950 (1994)
Wiringa *et al.*, PRC51, 38 (1995)
Machleidt, PRC63, 024001 (2001)
Entem *et al.*, PRC68, 041001 (2003)
Holt et al. PRC81, 024002 (2010)

t_{NN} above pion-production threshold



Bare $NN \to N + A$ connection

$$\underbrace{\mathbf{NN} \to \left\{ \begin{array}{c} \left(\frac{d\sigma}{d\Omega}\right)_{NN} \\ \downarrow\uparrow \\ \delta_L(E) \end{array} \right\}}_{\text{2-Body}} \rightleftharpoons \underbrace{V_{NN} \begin{array}{c} \nearrow \\ \searrow \end{array} \begin{array}{c} \mathscr{G}_{NN} \\ \\ t_{NN} \end{array} \Bigg\}}_{\text{Effective Interaction}} \to \underbrace{\underbrace{U_{opt}(\mathbf{k}',\mathbf{k};E)}_{\uparrow\,|\Psi_{g.s.}\rangle} \to \left\{ \begin{array}{c} |\psi\rangle \\ \left(\frac{d\sigma}{d\Omega}\right)_{NA} \\ \sigma(E) \end{array} \right.}_{\text{N+A}}$$



$$U(\mathbf{k}',\mathbf{k};E) = \int d\mathbf{p}\, d\mathbf{p}' \underbrace{\rho(\mathbf{p}',\mathbf{p})}_{HFB}\, \eta\, \underbrace{\langle \mathbf{k}'\,\mathbf{p}'\,|\,T(E)\,|\,\mathbf{k}\,\mathbf{p}\rangle}_{V_{NN}} \to \int d\mathbf{P}\, \underbrace{\rho(\mathbf{P}-\tfrac{\mathbf{q}}{2},\mathbf{P}+\tfrac{\mathbf{q}}{2})}_{g.s.}\, \eta\, \langle \mathbf{k_r}'\,|\,t(\sqrt{s})\,|\,\mathbf{k_r}\rangle$$

$$(\hat{K} + \hat{U}(E)\,)\,|\,\Psi\rangle = E\,|\,\Psi\rangle$$

Kinematics

Ref: PRC66, 024602 (2002)

Moller factor:

$$\eta(\mathbf{k}'\mathbf{p}';\mathbf{k}\,\mathbf{p}) = \left[\frac{\omega(\mathbf{k_r}')\varepsilon(-\mathbf{k_r}')\omega(\mathbf{k_r})\varepsilon(-\mathbf{k_r})}{\omega(\mathbf{k}')\,\varepsilon(\mathbf{p}')\,\omega(\mathbf{k})\,\varepsilon(\mathbf{p})}\right]^{1/2} ,$$

Relative momenta with mimimal relativity: Aaron-Amado-Young [Phys. Rev. **174**, 2022 (1968)] or Giebink [PRC **32**, 502 (1985)]

$$\mathbf{k_r} = W\mathbf{k} - (1-W)\mathbf{p} \qquad \mathbf{k_r}' = W'\mathbf{k}' - (1-W')\mathbf{p}'$$

$$W = \frac{\varepsilon + \varepsilon_r}{\varepsilon + \varepsilon_r + \omega + \omega_r} \quad \xrightarrow{NR} \quad \frac{m_N}{m_N + m_K}$$

$$\omega_r = \sqrt{m_K^2 + k_r{}^2}\,, \quad \varepsilon_r = \sqrt{m_N^2 + k_r{}^2}\,, \quad k_r{}^2 = \frac{1}{4s_{\rm in}}\xi^2(s_{\rm in}, m_K^2, m_N^2)\,,$$

where ξ corresponds to the Källen function

$$\xi(x,y,z) = \sqrt{(x-y-z)^2 - 4yz}$$

$$s_{\rm in} = (\varepsilon + \omega)^2 - (\mathbf{p} + \mathbf{k})^2$$

In this work... PRC76, 014613 (2007)

$$U_A(\mathbf{k}',\mathbf{k};\omega) = \sum_\alpha \int d\mathbf{P}\,\hat{\rho}_\alpha(\mathbf{q};\mathbf{P})\,\mathbf{t}_{Free}(\omega+\varepsilon_\alpha) - \frac{1}{6\pi^2}\int d\mathbf{Q}\,d\mathbf{P}\times$$
$$\sum_\alpha \hat{\rho}_\alpha(\mathbf{Q};\mathbf{P})\int_0^\infty z^3 dz\left[\frac{3j_1(z|\mathbf{Q}-\mathbf{q}|)}{z|\mathbf{Q}-\mathbf{q}|}\right]\frac{\partial g[\rho(z);\omega+\varepsilon_\alpha]}{\partial z}$$

Central plus spin-orbit

$$U(\mathbf{k}',\mathbf{k};\omega) = U_c(\mathbf{k}',\mathbf{k};\omega) + i\sigma\cdot\hat{n}\,U_{so}(\mathbf{k}',\mathbf{k};\omega)$$
$$= \sum_{\ell jm}\mathscr{Y}^m_{j\ell 1/2}(\hat{k}')\,U_{j\ell}(k',k;\omega)\,\mathscr{Y}^{m\dagger}_{j\ell 1/2}(\hat{k})$$

In r-space...

$$r'U_{j\ell}(r',r)r = \frac{2}{\pi}\int k^2 dk\,k'^2 dk'\,j_l(k'r')\,U_{j\ell}(k',k,\omega)\,j_l(kr)$$

NN bare potential with separable term

NN potential $\longrightarrow$ ‘Reference’ plus ‘separable’

$$V_{NN}(E) = V_{Ref} + |\xi\rangle\Gamma(E)\langle\xi|$$

$$\Gamma(E) \rightarrow \begin{cases} \text{energy dependent} \\ \text{state dependent} \\ \text{complex} \end{cases}$$

- $\Gamma(E)$ $\rightarrow$ match to the continuous-energy-fit solutions of $S-$matrices based on phenomenological complex phase shifts $\{\delta, \rho\}$ from the CNS Data Analysis Center at GWU (Richard Allen Arndt): INS DAC http://gwdac.phys.gwu.edu

On the separable strength...

For a given (known) realistic interaction ... say AV18

$$t_{18}(E) = v_{18} + v_{18}\, G_\circ(E)\, t_{18}(E)$$ known solution

$$V(E) = v_{18} + |\,\xi\rangle\lambda(E)\langle\xi\,|$$ add up rank-1 separable

$$T(E) = V(E) + V(E)\, G_\circ(E)\, T(E)$$ T matrix for the sum

Exact solution:

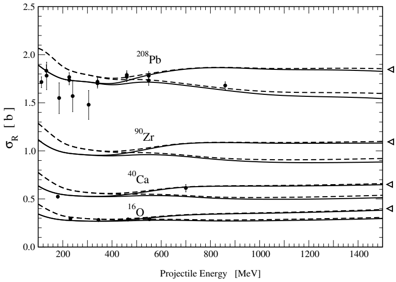
$$T(E) = t_{18}(E) + \frac{(1+t_{18}\, G_\circ)|\,\xi\rangle\lambda(E)\langle\xi\,|(1+G_\circ\, t_{18})}{1-\lambda(E)\langle\xi\,|(G_\circ + G_\circ\, t_{18}\, G_\circ)|\,\xi\rangle}$$

Project on-shell, constrain to data and solve for $\lambda(E)$.

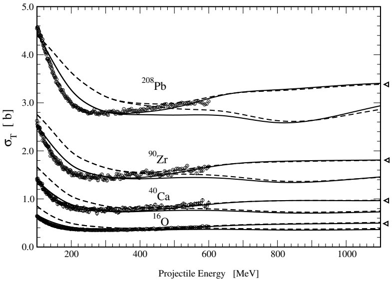
In-medium: $t \to g$; $G_0 \to G_{BBG}$

 Ref: PRC66, 024602 (2002)

Reaction cross section ($p \to A$):



Total cross section ($n \to A$):

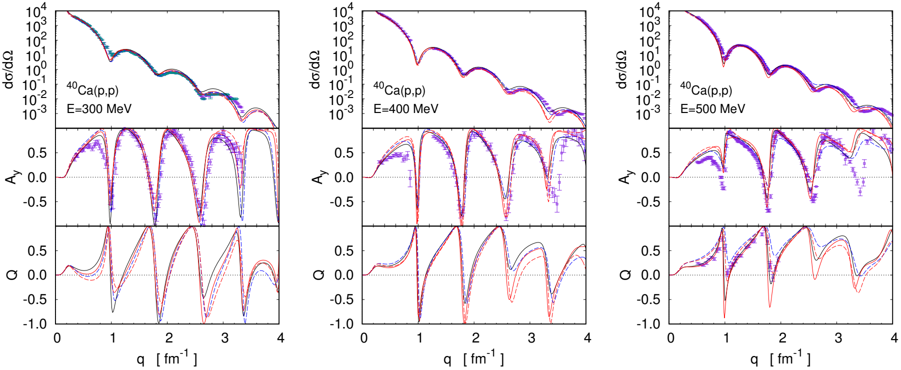


Key:

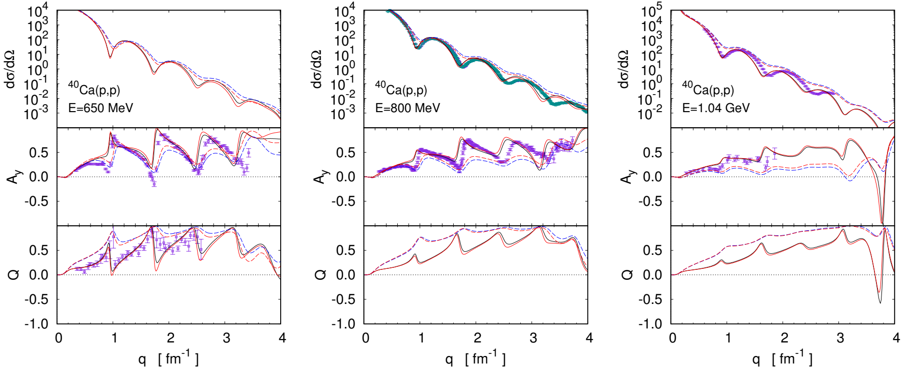
In-medium g-matrix (solid)
Free t matrix (dashed)

$\triangleleft \to$ full $\Gamma(E)$ (vs Re $\{\Gamma(E)\}$ only)

g vs t matrix; with vs without $|\xi\rangle\Gamma\langle\xi|$

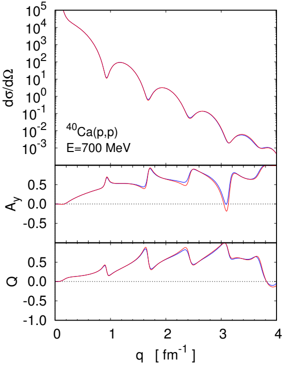


Solid curves (full Γ); Dashed curves ($\mathrm{Im}\{\Gamma\}=0$).
Red curves (g matrix); blue curves (t matrix)



Solid curves (full Γ); Dashed curves ($\mathrm{Im}\{\Gamma\} = 0$).
Red curves (g matrix); blue curves (t matrix)

Exact mixed density vs Slater approximation

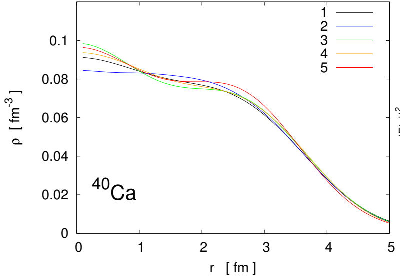


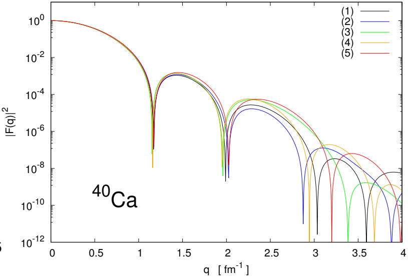
- Full mixed density:

$$\rho(p', p) = \sum_{occ} \varphi^{\dagger}(p')\varphi(p)$$

- Slater approximation: PRC42, 652(1990)
$\rho(z) = \hat{k}^3(z)/3\pi^2$

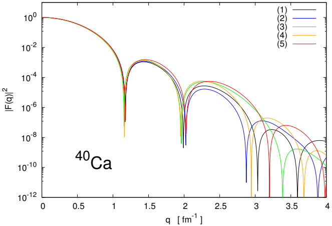
$$\rho(P + \tfrac{q}{2}, P - \tfrac{q}{2}) = \tfrac{1}{\pi^2} \int_0^{\infty} z^2 dz j_0(qz) \Theta[\hat{k}(z) - P]$$

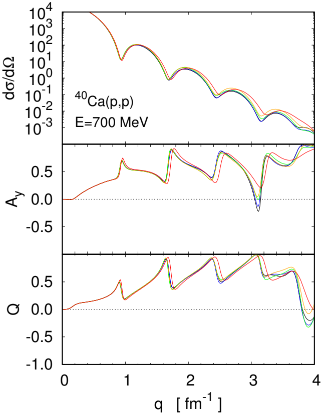




Set	Rp	Rn	Rm	Rch	Id
1 ●	3.448	3.395	3.422	3.539	Negele-70
2 ●	3.423	3.382	3.403	3.516	RPA
3 ●	3.405	3.365	3.385	3.497	HFB+D1S (Girod)
4 ●	3.400	3.356	3.378	3.493	HF
5 ●	3.372	3.300	3.336	3.466	Toy

p scattering





$q = 2q_0 \sin(\theta_{cm}/2)$
$q \sim (1.5 - 3)\ \text{fm}^{-1}$
$q_0 \approx 6.5\ \text{fm}^{-1}$
$\theta_{cm} \sim (13 - 27)$ deg ?)

Summary and up next...

- Inclusion of *NN* absorption important to describe NA scattering at $E > 500$ MeV
- Reasonable description of cross sections and polarization observables
- Investigated sensitivity of proton-calcium scattering to the proton/neutron density at 700 MeV $\rightarrow$ room to set bounds on nuclear matter distribution: $\rho(z) = \rho_0(z)[1+\beta^2(z)]$ NPA 755, 527c (2005)

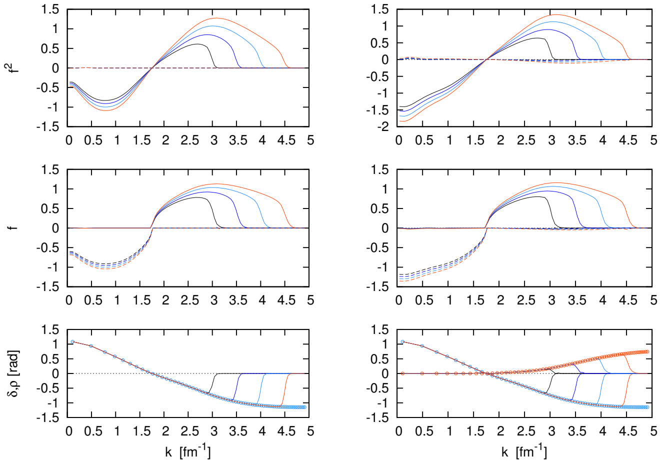
Ongoing...

- Infer $\beta(z)$ from $(d\sigma/d\Omega)_{exp}$
- Energy-independent separable supplement to reproduce *NN* scattering amplitudes up to 1.2 GeV (Master's thesis U Chile - Nelson Adriazola) $V_{NN} = V_{Ref} + |\xi\rangle\langle\xi|$

Thanks ...

Backup slides

Sensitivity of the separable form factor in momentum space for the state $^1S_0$ for different choices of the cut-off parameter



Separable form factors in coordinate space for spin-uncoupled states in the case $J \leq 2$