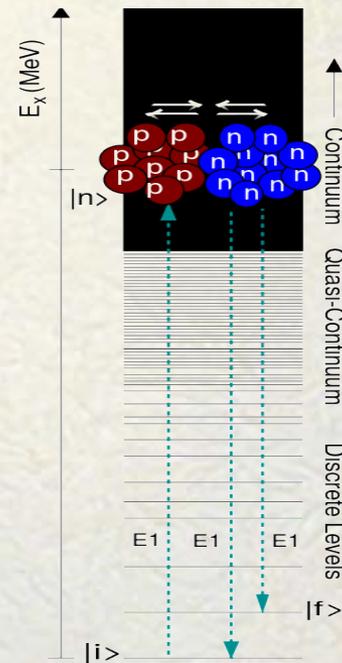
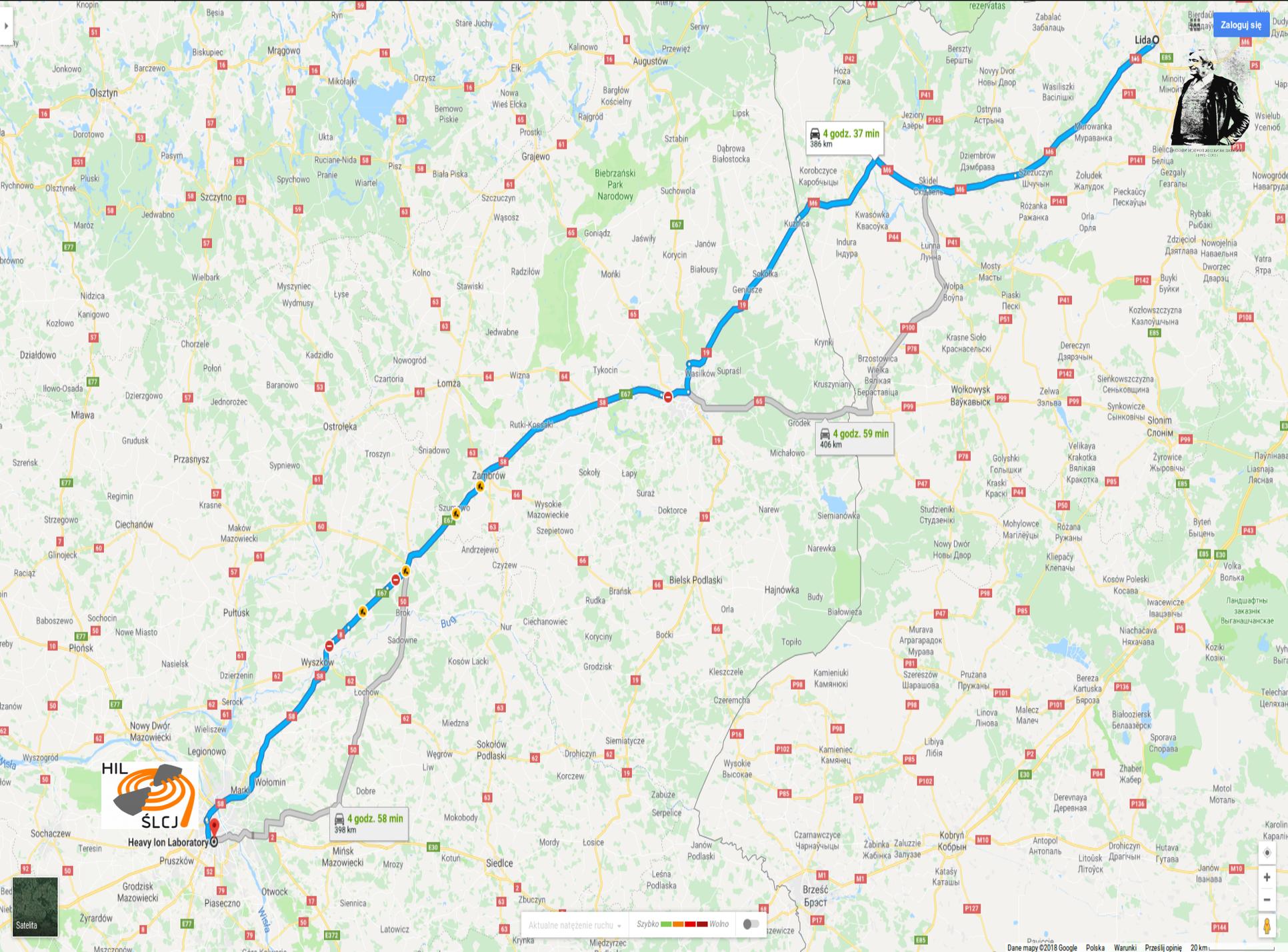


Nuclear polarizability effects in Coulomb-excitation studies of light and medium-mass nuclei



ARKADIÏ BENEDIKTOVICH MIGDAL
(1911–1991)





4 godz. 37 min
386 km

4 godz. 59 min
406 km

4 godz. 58 min
398 km



Heavy Ion Laboratory

Aktualne natężenie ruchu - Szybko [color scale] Wolno [color scale]

Coulomb excitation of light nuclei

Problems with Light Nuclei

Adiabaticity

Safe energies

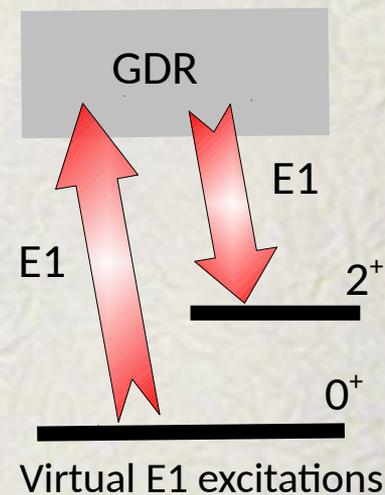
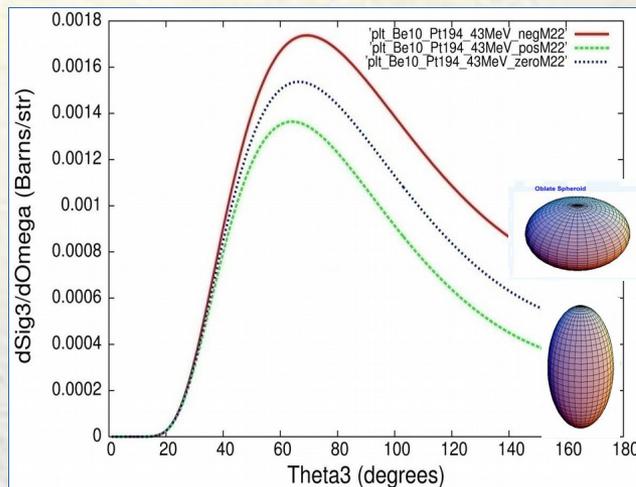
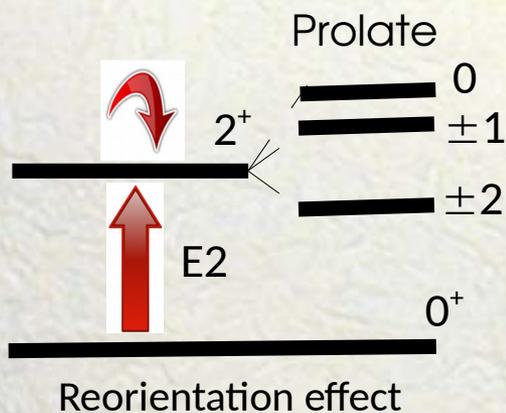
Quantal effects

Polarizability

Coulomb-excitation Perturbation Theory

Polarizability (another 2nd order effect)

$$\sigma_{E2} = \sigma_R [k_1(\theta_{CM}, \xi) B(E2) (1 + k_2(\theta_{CM}, \xi) Q_S(2_1^+))]]$$

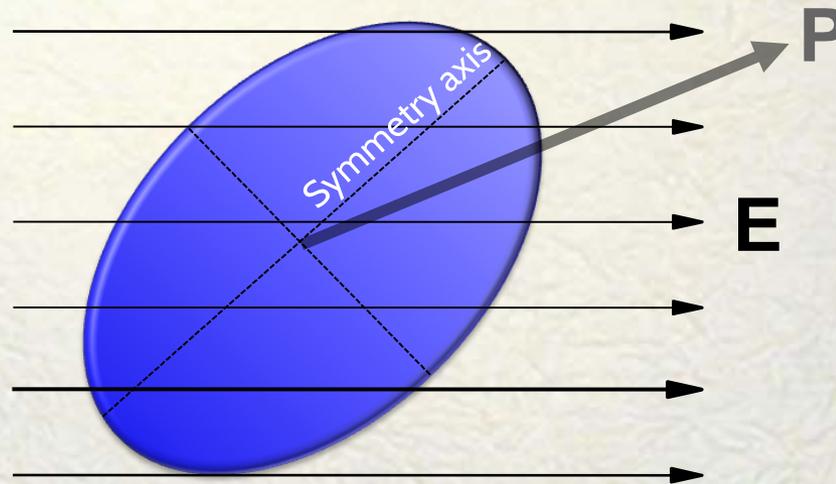


Virtual excitations via the GDR may affect $B(E2)$ and Q_S values

J. Eichler, Phys. Rev. **133**, B1162 (1964)

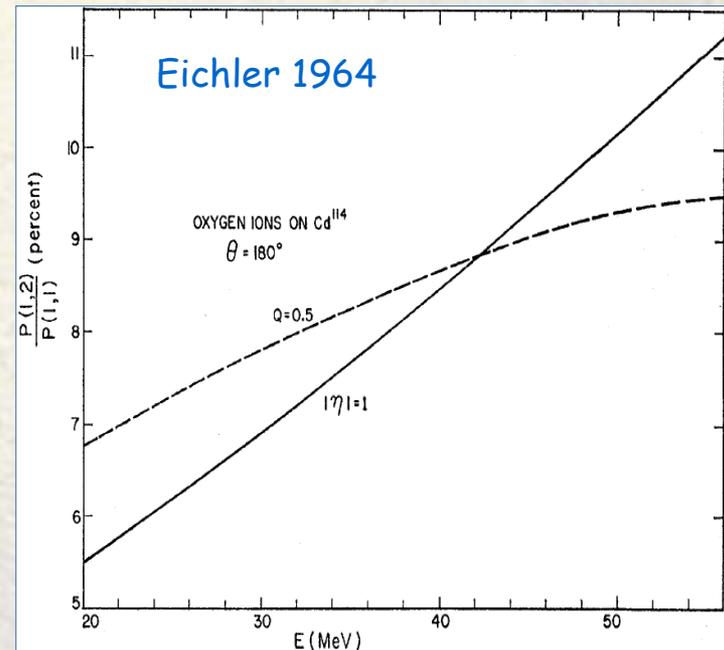
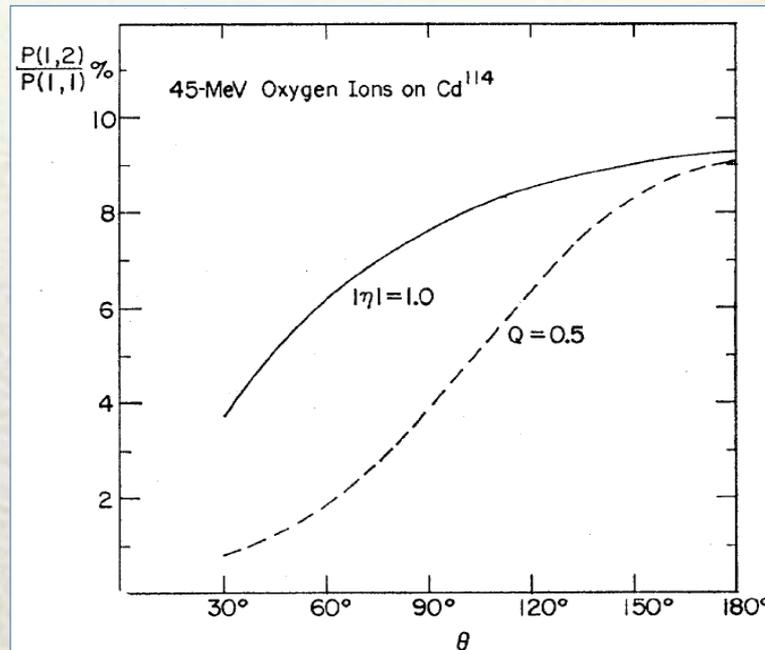
$$P = \alpha E$$

The torque produced by the interaction between E and P may set the nucleus into rotation \rightarrow **enhancement of quadrupole collectivity**



Deformed nucleus in an external homogeneous electric field, E

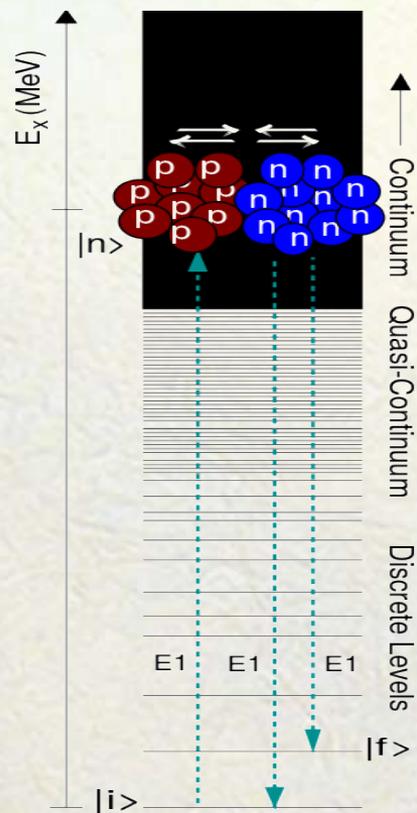
Disentangling Nuclear Polarizability and Reorientation Effects through Coulomb excitation @ different angular ranges and bombarding energies



The difference in the angular behaviour or in the energy dependence of second-order effects in Coulomb excitation may disentangle the two effects

Nuclear Polarizability (another second order effect)

High impact in spectroscopic quadrupole moment measurements of light nuclei



$$\alpha = 2e^2 \sum_n \frac{\langle i \| \hat{E}1 \| n \rangle \langle n \| \hat{E}1 \| i \rangle}{E_\gamma} = \frac{\hbar c}{2\pi^2} \sigma_{-2}$$

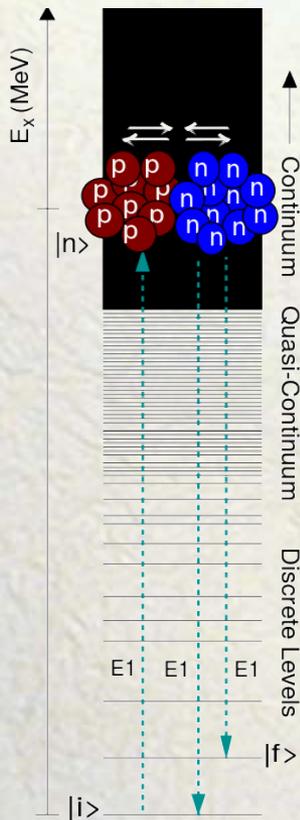
Large E1 matrix elements via virtual excitations of the GDR may polarize the shape of the ground and excited states

Virtual excitations via the GDR may affect $B(E2)$ and Q_s values

J. Eichler, Phys. Rev. **133**, B1162 (1964)

Nuclear Polarizability, α

A B Migdal introduced implicitly the concept of a dynamic collective model in nuclear physics and used this concept to predict a giant dipole resonance



Hydrodynamic Model

$$\alpha = \frac{e^2 R^2 A}{40 a_{\text{sym}}}$$

Second-order Perturbation Theory

$$\alpha = 2e^2 \sum_n \frac{\langle i || \hat{E}1 || n \rangle \langle n || \hat{E}1 || i \rangle}{E_\gamma} = \frac{\hbar c}{2\pi^2} \sigma_{-2}$$

$$\sigma_{-2} = 2.25 A^{5/3} \mu\text{b}/\text{MeV}$$

$$R = r_0 A^{1/3} \text{ fm} \quad a_{\text{sym}} = 23 \text{ MeV}$$

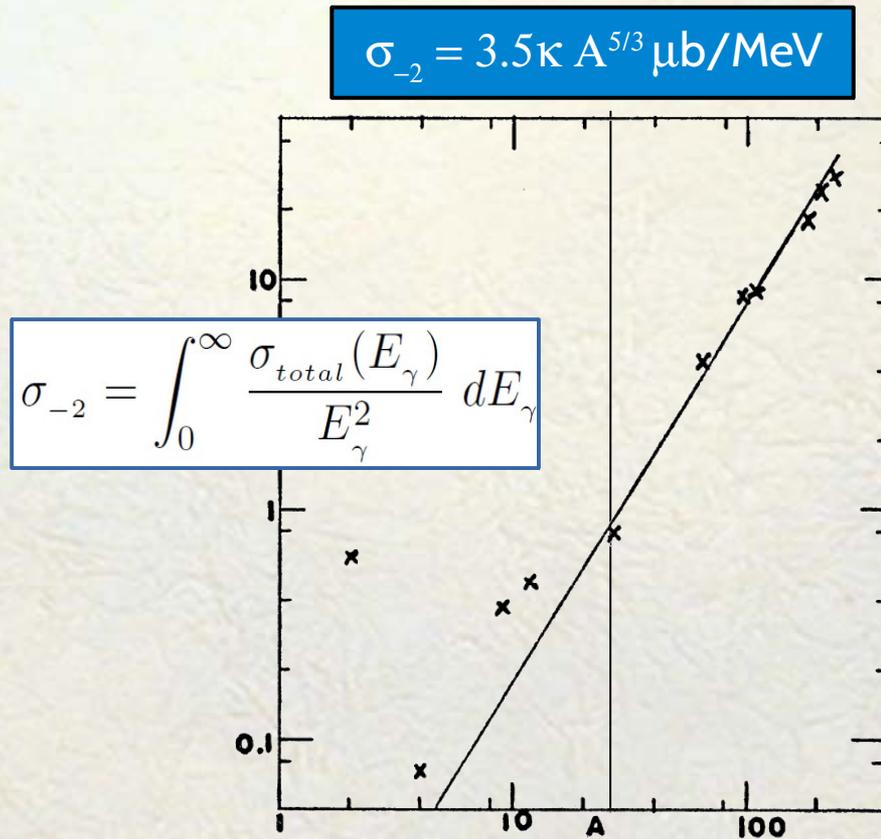


АРКАДИЙ БЕНЕДИКТОВИЧ МИГДАЛ
(1911-1991)

A.B. Migdal, J. Exp. Theor. Phys. USSR 15, 81 (1945)

Nuclear Polarizability, α

Levinger confirmed Migdal's power law from available photo-absorption data



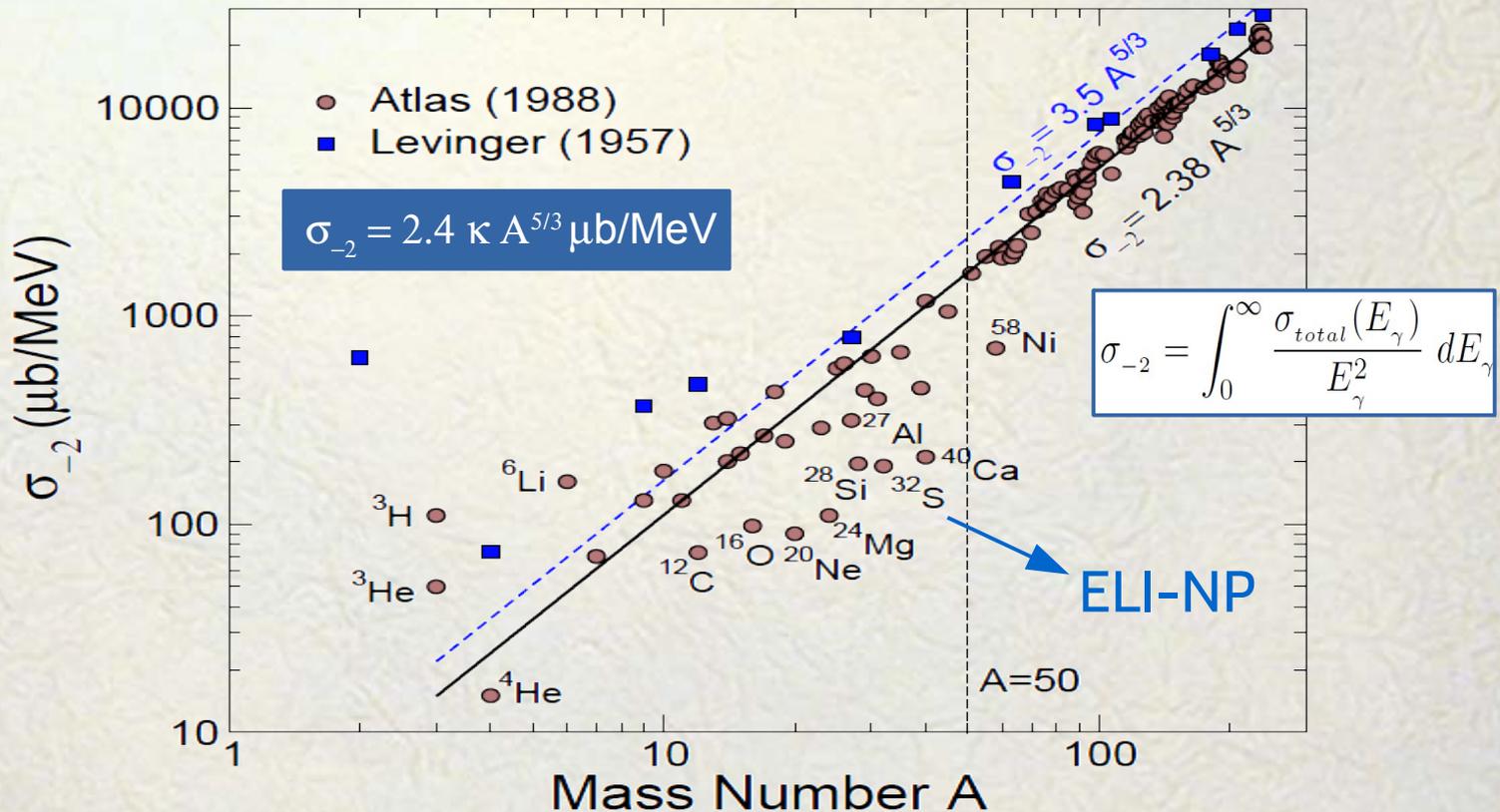
The κ polarizability parameter: [deviations from the GDR effects for light nuclei](#)

J. S. Levinger, Phys. Rev. **107**, 554 (1957)

New power-law formula for σ_{-2}

Dietrich & Berman photoneutron cross-section evaluation (1988)

J. N. Orce, PRC (2015), Comment (von Neunman-Cosel) (2016) + Reply (2016)

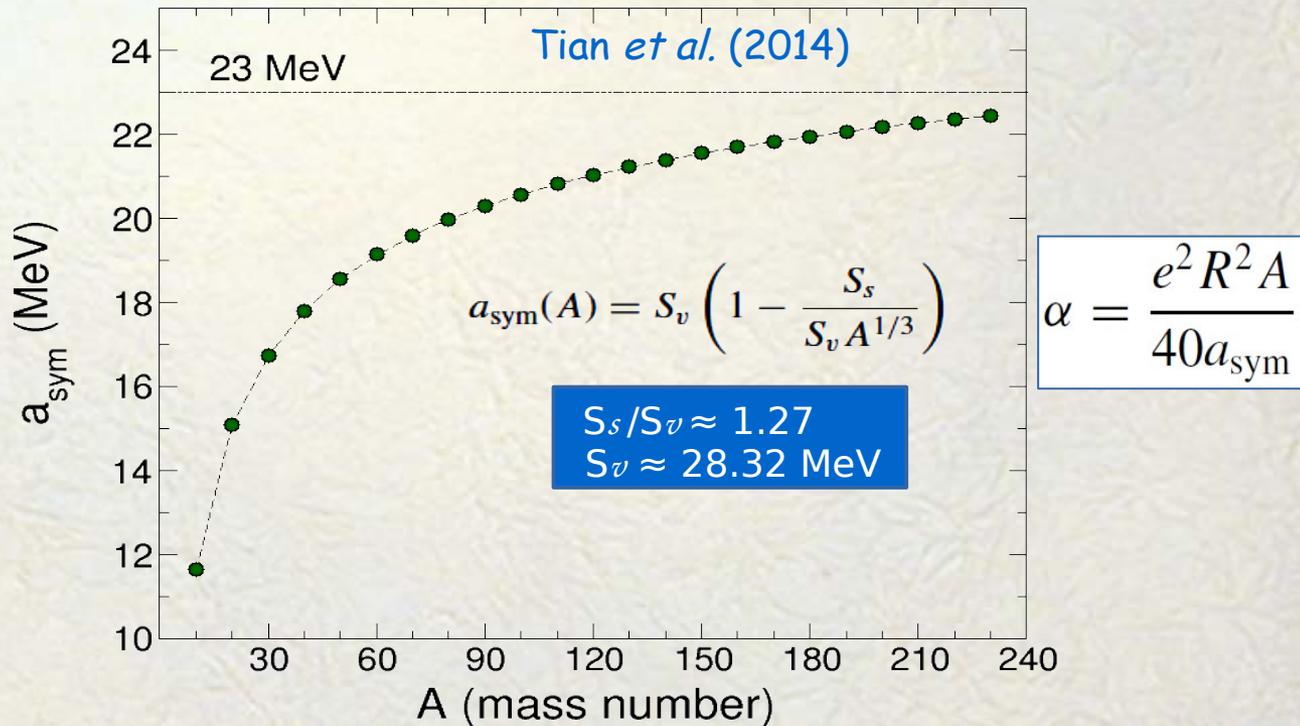


$\kappa > 1$ for light loosely bound nuclei (also from Coulex: ${}^6\text{Li}$, ${}^7\text{Li}$, ${}^{17}\text{O}$)

$\kappa < 1$ for $T_z = 0$ self-conjugate nuclei: missing (γ, p) contribution (Morinaga)

a_{sym} is not 23 MeV: $a_{\text{sym}}(A)$

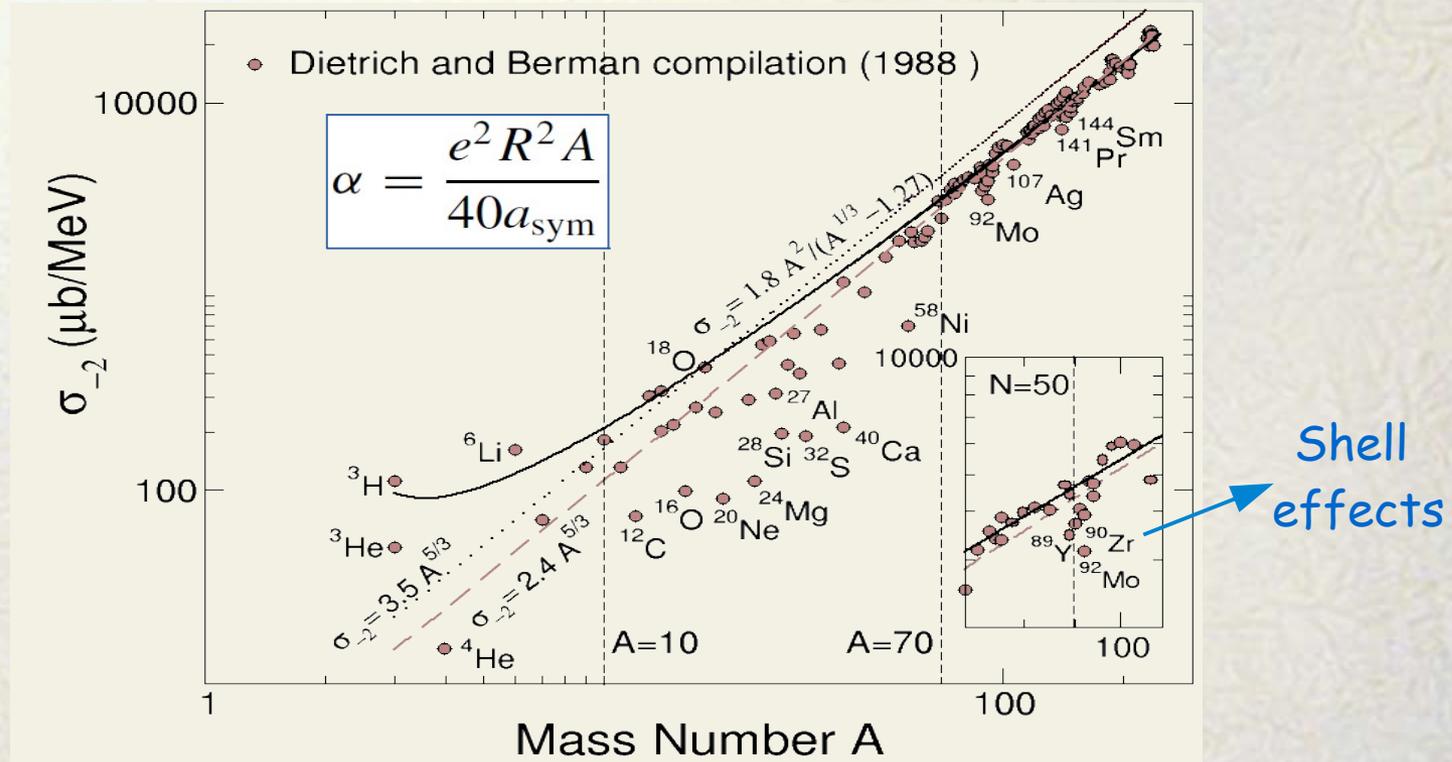
From a global fit to the binding energies of isobaric nuclei with $A \geq 10$, extracted from the 2012 atomic mass evaluation



The nuclear symmetry energy, $a_{\text{sym}}(A)$, is key to understanding the elusive equation of state of neutron-rich matter, which impacts three-nucleon forces, neutron skins, neutron stars and supernova cores

New formula for σ_{-2} from $a_{\text{sym}}(A)$

The increasing upbend observed as A decreases provides an explanation for the large GDR effects observed in light nuclei. Both merge with data at $A \sim 70$

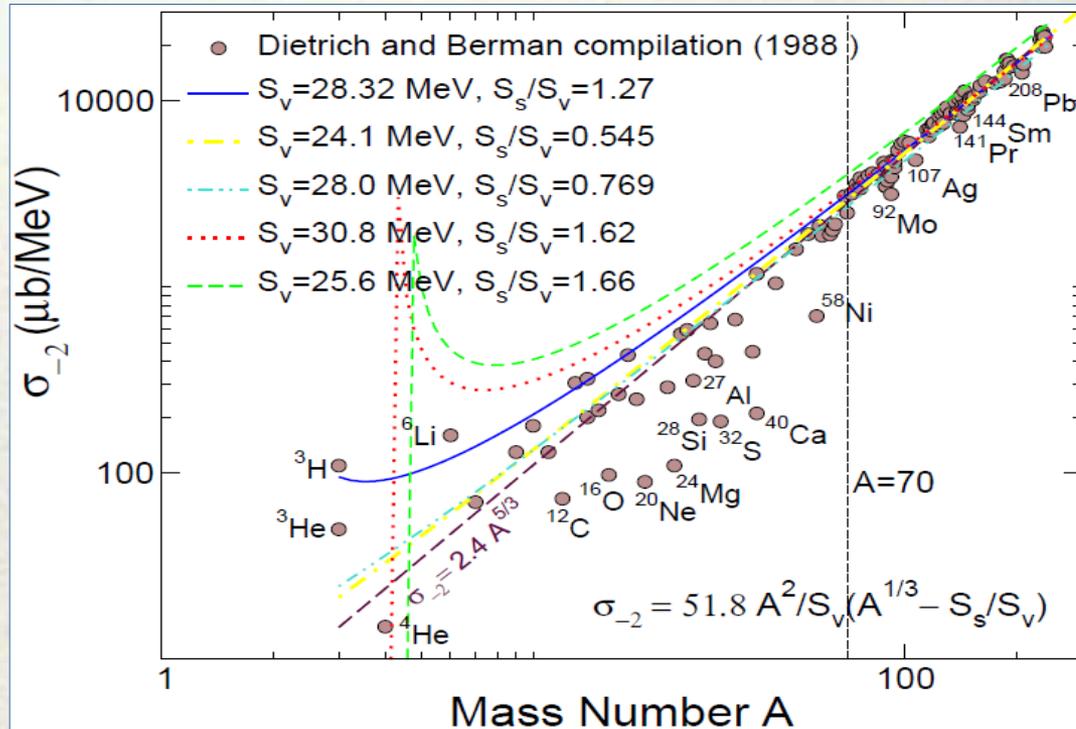


$\kappa > 1$ for light loosely bound nuclei, J. N. Orce, PRC (2015)

The validity of the hydrodynamic model to be tested for the lightest $A < 10$ nuclei

Constraining $a_{\text{sym}}(A)$

High-precision relativistic (p,p') measurements of α by von Neumann-Cozel *et al.*, high-energy, binding energies, total photo-absorption cross sections and IVGDRs



J. N. Oorce, Proc. of the 4th South Africa - JINR Symposium (2016)
P. von Neunman-Cosel PRC (2016) + J. N. Oorce, PRC (2016)

Polarization potential reduces the effective quadrupole potential

K. Alder, A. Winther - Electromagnetic Excitation (1975) – Appendix J

$$\begin{aligned} V_{eff}(t) &= V_0(t) (1 - V_{pol}(t)) \\ &= V_0(t) \left(1 - z \frac{a}{r(t)} \right). \end{aligned}$$

default value in GOSIA

$$z = \frac{10Z_t\alpha}{3Z_pR^2a} \approx 0.005 \frac{E_p A_p}{Z_p^2(1 + A_p/A_t)} \quad \text{for } \kappa=1$$

$$\alpha = \frac{\hbar c}{4\pi^2} \sigma_{-2} \quad \sigma_{-2} = 3.5 \kappa A^{5/3} \mu\text{b/MeV}$$

Adjustable empirical E1 polarization strength (Häusser, Vermeer (1960-70s))

New polarization potentials change this slightly

GOSIA Code

DIPOL - E1 polarization parameter

DIPOL = 0.005 (using Levinger's power-law formula)

ZPOL - dipole term (GDR excitation)

$$z(1, 1, 2) = \frac{10Z_1P_0}{3Z_2R_{qa}^2} \sim 0.5 \times 10^{-2} \frac{E_{MeV}A_2}{Z_2^2(1 + A_1/A_2)}$$

Nigel's gosia page

This is **not** the official gosia web page. This page has various versions of gosia, most of which have not been tested. They are intended to allow users who find bugs, to track down which versions have the bug and which don't.

The official gosia home page is in Rochester at: <http://www.pas.rochester.edu/~cline/Gosia/index.html>. There you will find the most recent versions of gosia, gosia2, annl, sigma, select, and gremlin. and the manual, which Doug Cline is maintaining.

The Warsaw group also has a gosia page with some talks by Magda and Kasia to introduce beginners to Gosia: <http://www.slj.uw.edu.pl/en/81.html>. It also has a link to the web page for the first gosia workshop (April 2008) in Warsaw: http://www.slj.uw.edu.pl/gosia_workshop which has the presentations made at that workshop.

Here you can find various versions of gosia. They should (in theory) be similar in the way they work, but if you find a bug in the most recent version, please test some of the older versions to help track down where the bug was introduced. The versions have a date stamp YYYYMMDD and possibly a sub-version. Sub-versions are for bugfixes only (relative to the major version with the date stamp).

Gosia

For target OR projectile excitations, separately.

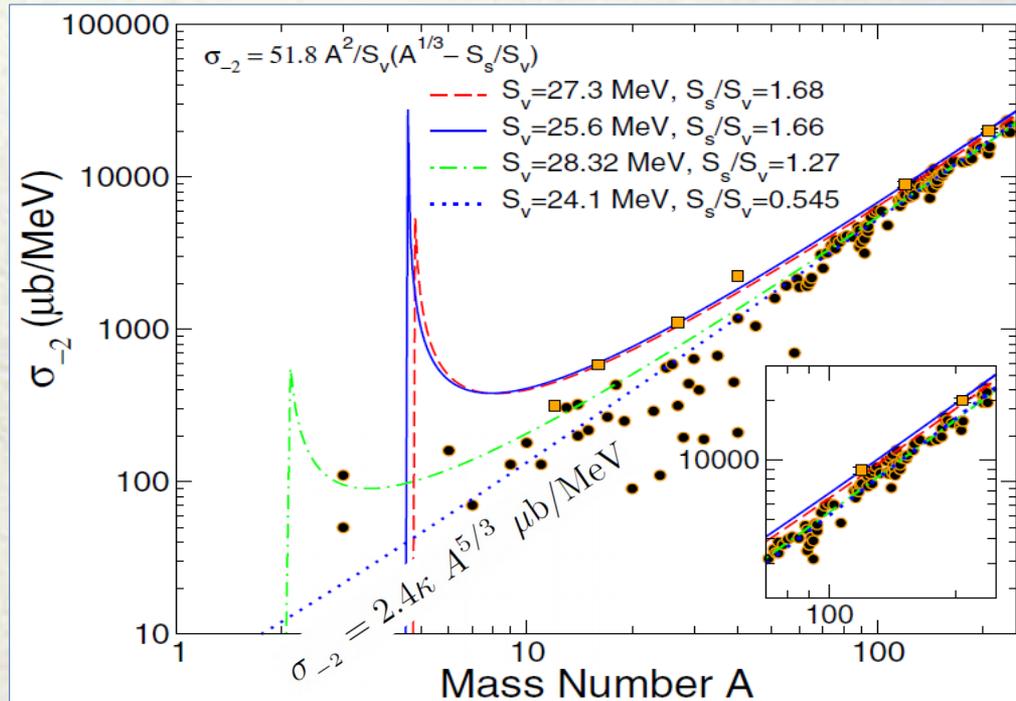
Date	File	Comments
28 Nov 2016	gosia_20110524.5.f	Clean up some warnings by explicitly converting types.
12 Feb 2016	gosia_20110524.4.f	Allow different efficiency parameters for different experiments.
7 Mar 2014	gosia_20110524.3.f	Bugfix relative to 20110524.2 (Fix bug in determination of IKIN in OP,INTI).
7 Nov 2011	gosia_20110524.2.f	Bugfix relative to 20110524.1 (Fix bug in lambda=6 mu=3 collision functions).
10 Oct 2011	gosia_20110524.1.f	Bugfix relative to 20110524 (Fix bug which causes a failure when a meshpoint in inverse kinematics is at the target angle corresponding to the maximum projectile angle).
24 May 2011	gosia_20110524.f	Version including extra options for Rachel.
7 Mar 2014	gosia_20081208.13.f	Bugfix relative to 20081208.12 (Fix bug in determination of IKIN in OP,INTI).
7 Nov 2011	gosia_20081208.12.f	Bugfix relative to 20081208.11 (Fix bug in lambda=6 mu=3 collision functions).
10 Oct 2011	gosia_20081208.11.f	Bugfix relative to 20081208.10 (Fix bug which causes a failure when a meshpoint in inverse kinematics is at the target angle corresponding to the maximum projectile angle).
24 June 2010	gosia_20081208.10.f	Bugfix relative to 20081208.9 (Fix bug in relativistic correction for inverse kinematics).
22 February 2010	gosia_20081208.9.f	Bugfix relative to 20081208.8 (Fix discontinuity in TRINT).
18 September 2009	gosia_20081208.8.f	Improvements relative to 20081208.7 (Increased dimensions in ZETA to allow 999 matrix elements correctly this time).
14 September 2009	gosia_20081208.7.f	Improvements relative to 20081208.6 (Increased dimensions in ZETA to allow 999 matrix elements).
16 August 2009	gosia_20081208.6.f	Bugfix relative to 20081208.5 (Increased dimensions in VLIN to 101).
20 July 2009	gosia_20081208.5.f	Bugfix relative to 20081208.4 (integration over PIN diodes was incorrect).
2 April 2009	gosia_20081208.4.f	Bugfix relative to 20081208.3 (E1 polarization was incorrect).
1 February 2009	gosia_20081208.3.f	Bugfix relative to 20081208.2 (OP,INTI preserves IKIN flag).
27 January 2009	gosia_20081208.2.f	Bugfix relative to 20081208.1 (OP,INTI got wrong particle for target excitations).
16 January 2009	gosia_20081208.1.f	Bugfix relative to 20081208 (give error if too many omega steps).
8 December 2008	gosia_20081208.f	New OP,INTI option for inverse kinematics (beta version) - see doc here .
7 November 2011	gosia_20080630.9.f	Bugfix relative to 20080630.8 (Fix bug in lambda=6 mu=3 collision functions).
24 June 2010	gosia_20080630.8.f	Bugfix relative to 20080630.7 (Fix bug in relativistic correction for inverse kinematics).

New polarization potentials

incorporated in GOSIA: nuclear polarizability κ for deviations from GDR effect

$$V_{eff}(t) = V_0(t) (1 - V_{pol}(t)) = V_0(t) \left(1 - z \frac{a}{r(t)}\right)$$

$$\sigma_{-2} = 2.4\kappa A^{5/3} \mu\text{b/MeV} \rightarrow z = \frac{10Z_t \alpha}{3Z_p R^2 a} \approx 0.0039 \kappa \frac{T_p A_p}{Z_p^2 (1 + A_p/A_t)}$$



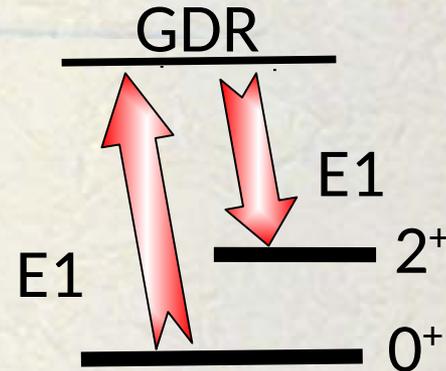
The nuclear polarizability κ from Coulex & SM calculations

Häusser, Barker, Navratil

GDR Contribution to Coulomb Excitation. I 1p Shell Nuclei

F. C. Barker Aust. J. Phys. (1982)

Department of Theoretical Physics, Research School of Physical Sciences,
Australian National University, P.O. Box 4, Canberra, A.C.T. 2600.



$$X \equiv S(E1) / \langle i || \mathcal{M}(E2) || f \rangle,$$

$$S(E1) = \sum_n W(11I_i I_f, 2I_n) \langle i || \mathcal{M}(E1) || n \rangle \langle n || \mathcal{M}(E1) || f \rangle / (E_n - E_i)$$

$$X_0 = 0.00058 A/Z e \text{ MeV}^{-1}.$$

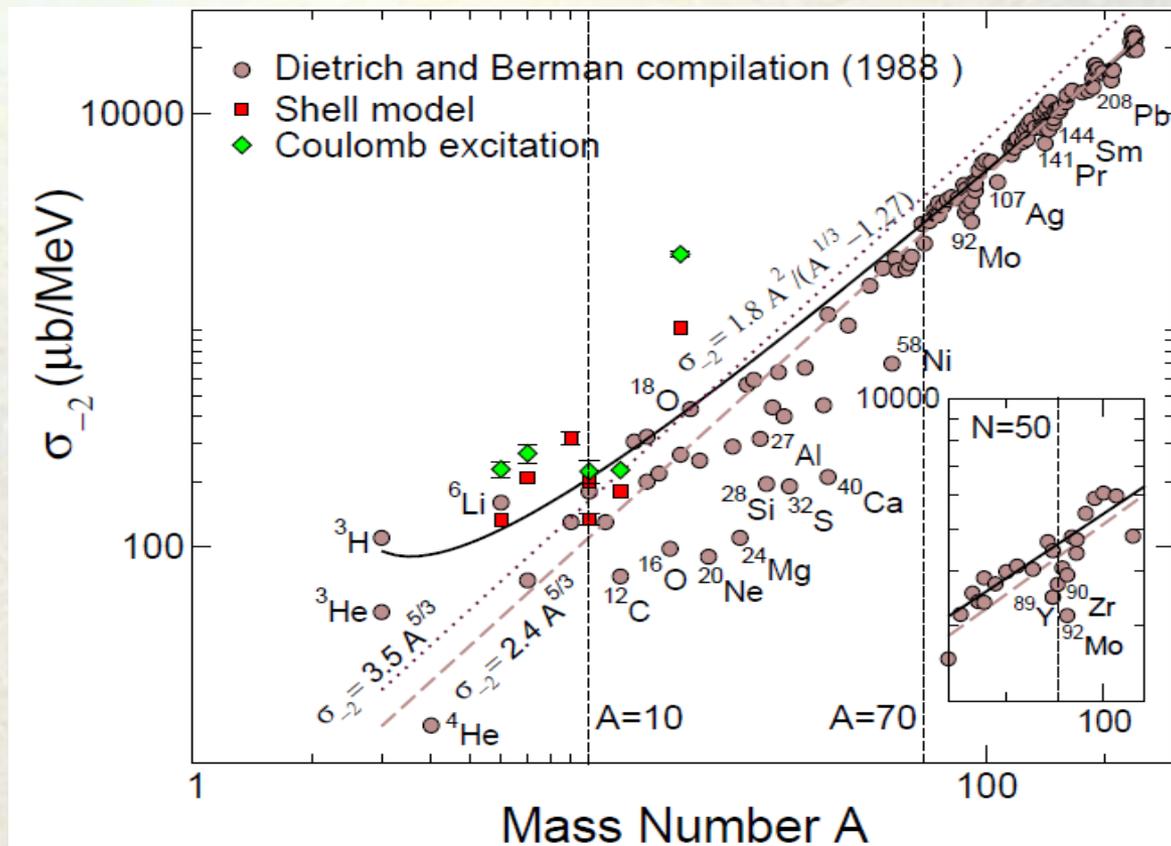
$$k = X/X_0$$

$$V_{\text{eff}}(t) = V_0(t) \left(1 - 9.6 \frac{E}{Z} \frac{S(E1)}{\langle i || \mathcal{M}(E2) || f \rangle} \frac{a}{r_p(t)} \right)$$

O. Häusser, NPA (1973)

Coulex measurements and shell model calculations of κ

Discrepant SM and Coulex Results



Virtual excitations via the GDR affect $B(E2)$ and Q_s values

k from SM calculations assuming that all the E1 strength from the ground state is concentrated at an energy E_{GDR}

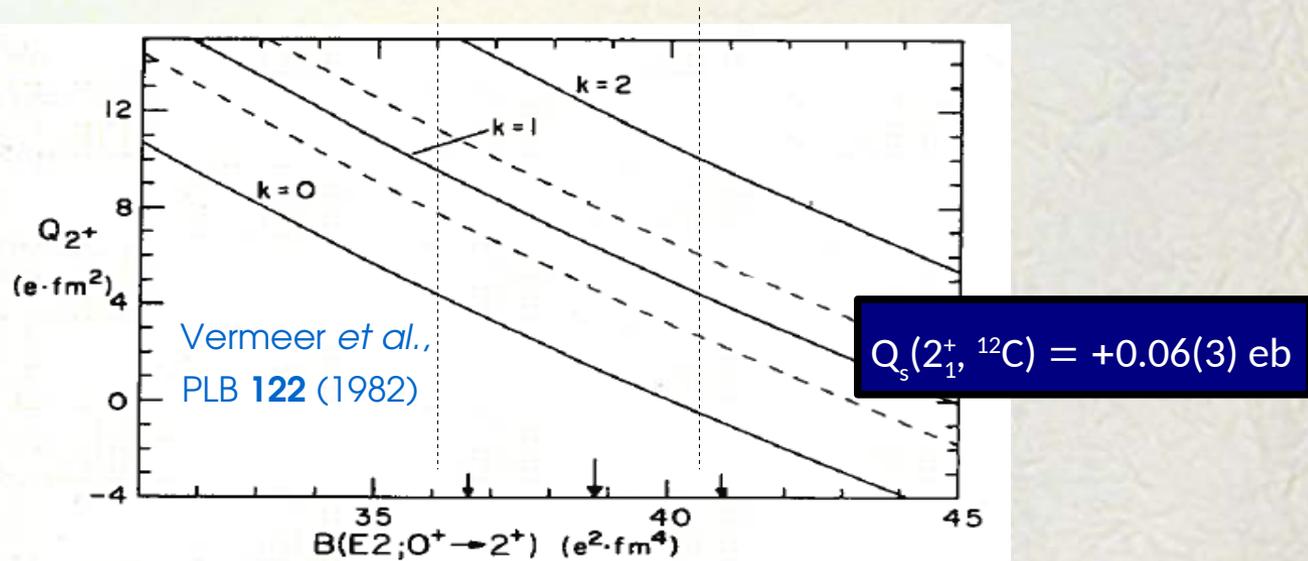


Fig. 3. Deduced values for Q_{2+} plotted as a function of the assumed value of $B(E2; 0^+ \rightarrow 2^+)$, for indicated values of k . Dashed curves indicate the experimental uncertainty for the case of $k = 1$. The arrows on the abscissa indicate the value and associated uncertainty of $B(E2; 0^+ \rightarrow 2^+)$ adopted in ref. [6].

J. N. Orce et al., PRC(R) (2012), Kumar Raju et al., PLB (2018)

New 2^+ polarizability from NCSM (chiral NN+3N / $N_{\text{max}}=6$ / up to 30 1^- states)

NCSM calculation of the nuclear polarizability κ using chiral NN+3N

- Model spaces with basis sizes of $N_{\max} = 4$ for natural and $N_{\max} = 5$ for unnatural parity states
- E1 matrix elements from all the transitions connecting 1^- states up to 30 MeV

$$\sigma_{-2} = 244 \mu\text{b}/\text{MeV}$$

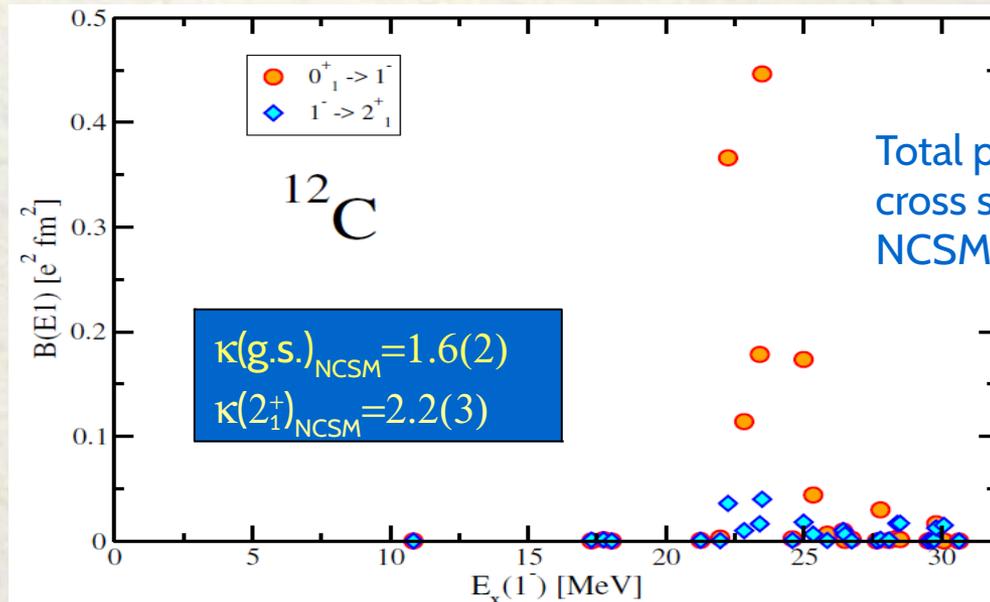
E. G. Fuller, Phys. Rep. **127**, 185 (1985)
Total photo-absorption cross section

$$\sigma_{-2} = 2.4\kappa A^{5/3} \mu\text{b}/\text{MeV}$$

J. N. Orce, Phys. Rev. C **91**, 064602 (2015)



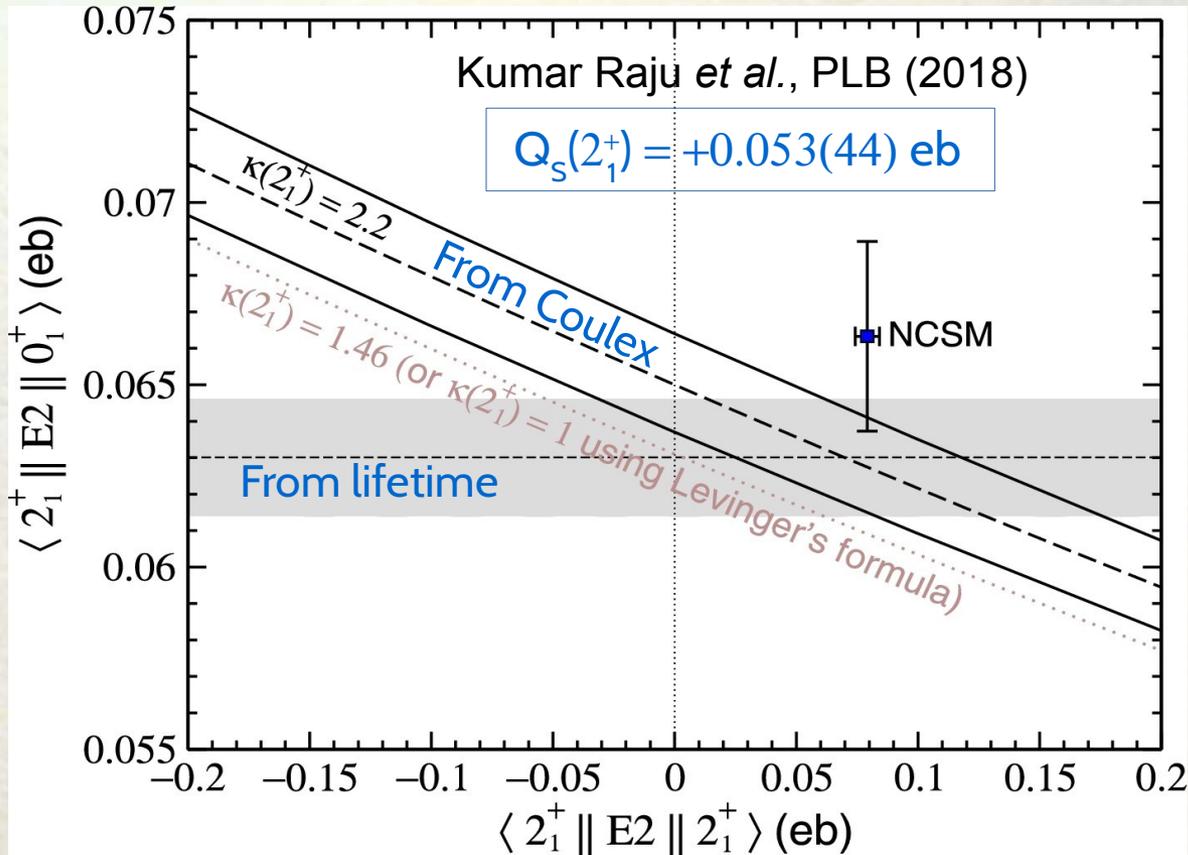
$$\kappa(\text{g.s.})_{\text{exp}} = 1.62$$



Kumar Raju *et al.*, PLB (2018)

Coulomb-excitation curve vs lifetime (normalization procedure)

$$Q_s(2_1^+) = 0.75793 \langle 2_1^+ \parallel E2 \parallel 2_1^+ \rangle$$



NCSM by C. Forssén, R. Roth and P. Navratil, J. Phys. G **40**, 055105 (2013)

B(E2; $0^+ \rightarrow 2^+$) value in ^{18}O

Long-standing discrepancy between Coulex and high-precision lifetime measurement

**A DBLA MEASUREMENT
OF THE LIFETIME OF THE FIRST EXCITED LEVEL IN ^{18}O
AND A CRITICAL COMPARISON OF LIFETIME RESULTS
BY DIFFERENT METHODS**

G. C. BALL, T. K. ALEXANDER, W. G. DAVIES, J. S. FORSTER and I. V. MITCHELL
*Atomic Energy of Canada Limited, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada
K0J1J0*

Received 26 November 1980

(Revised 22 July 1981)

Abstract: An accurate measurement of the lifetime of the 1.982 MeV level in ^{18}O has been obtained by bombarding ^4He and ^1H implanted targets with beams of 34 and 47 MeV ^{18}O ions. The mean lifetime determined by fitting the Doppler-broadened γ -ray lineshapes with experimentally determined stopping powers was 2.80 ± 0.07 ps. A detailed comparison of the mean lifetime values obtained by different techniques for this level and several other sd shell transitions suggest that recent Coulomb excitation $B(E2; 0^+ \rightarrow 2^+)$ values for ^{18}O are $\approx 10\%$ too small.

The remarkable case of ^{18}O

$B(E2; 0^+ \rightarrow 2^+)$ value in ^{18}O

TABLE IV. Integrated cross sections for ^{18}O (integrated up to 41.8 MeV).

Reaction	σ_{int} (MeV mb)	σ_{int} (sum-rule units)	σ_{-1} (mb)	σ_{-2} (mb MeV $^{-1}$)
$^{18}\text{O}(\gamma, p)$	44.4	0.17	1.66	0.064
$^{18}\text{O}(\gamma, n)$	121.5	0.46	5.93	0.342
$^{18}\text{O}(\gamma, 2n)$	76.7	0.29	3.14	0.141
$^{18}\text{O}(\gamma, n_t)$	198.3	0.74	9.08	0.483
$^{18}\text{O}(\gamma, \text{tot})$	242.6	0.91	10.73	0.547

$$\sigma_{-2} = 547 \mu\text{b/MeV} = 2.4 \kappa A^{5/3} \mu\text{b/MeV}$$



$$\kappa = 1.84$$

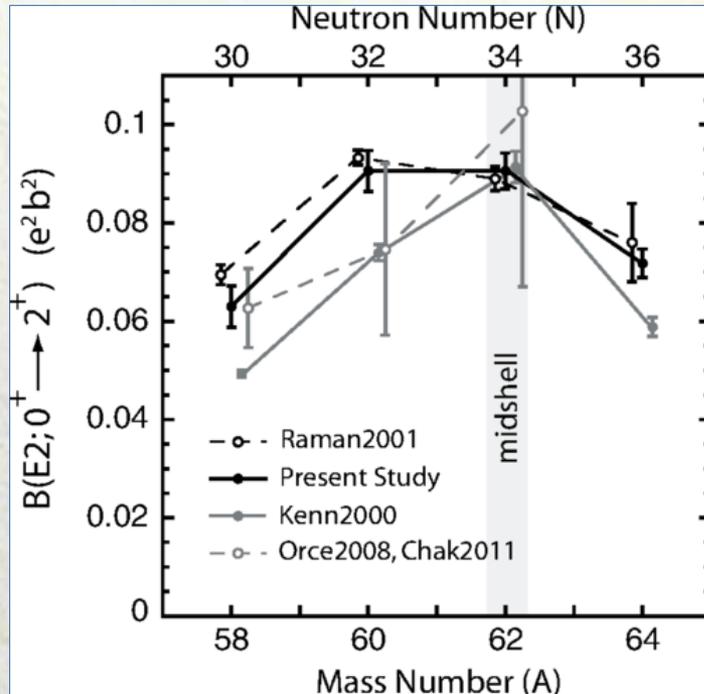
$$z = \frac{10Z_t \alpha}{3Z_p R^2 a} \approx 0.0039 \kappa \frac{T_p A_p}{Z_p^2 (1 + A_p/A_t)}$$

$B(E2)$ values underestimated in previous Coulex measurements by $\sim 10\%$!

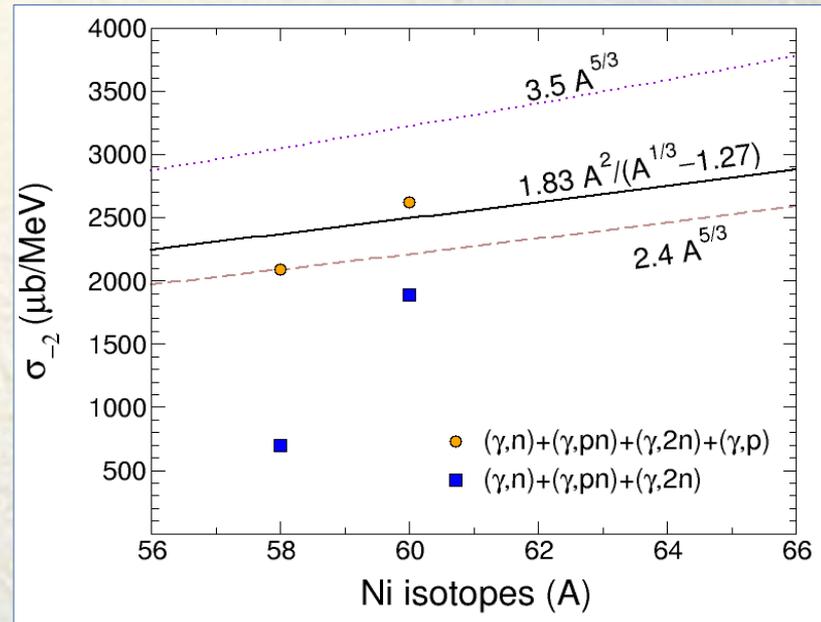
J. N. Orce, to be submitted

The remarkable case of the Nickels

Discrepancies between lifetimes vs Coulex in ^{58}Ni



J. M. Allmond *et al.*, PRC (2014)



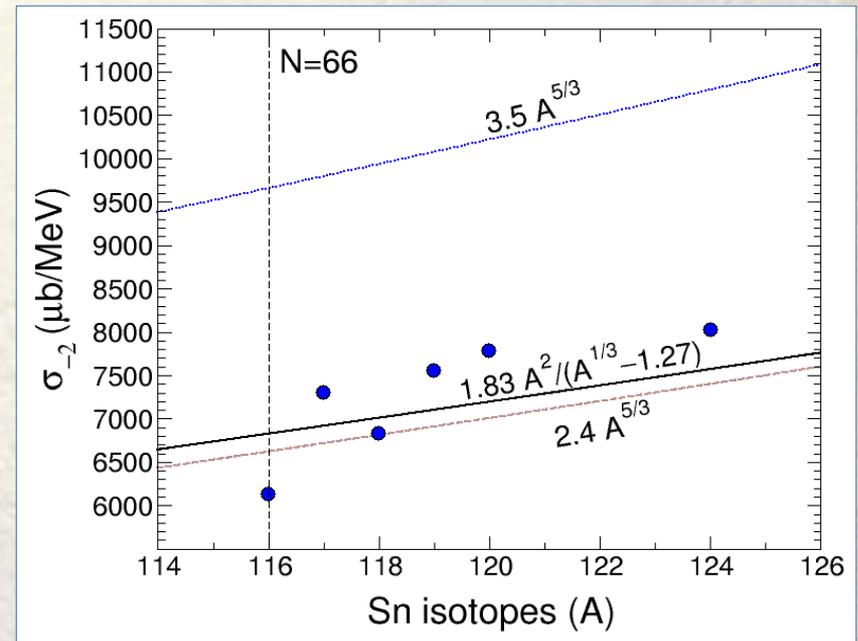
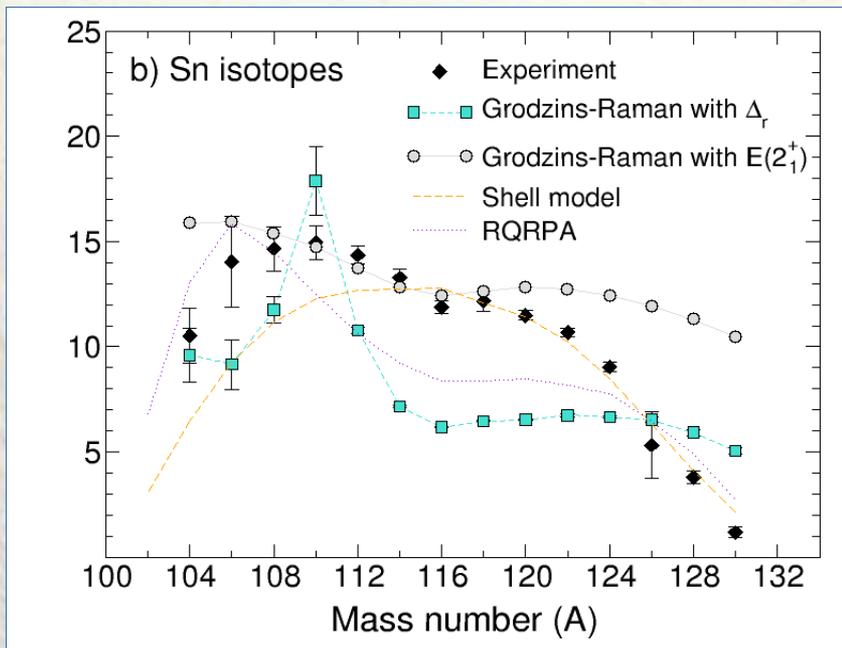
Everything is OK once you add the photoproton contributions.

A small $\kappa < 1$ do not enhance collectivity.

J. N. Orce, to be submitted

The remarkable case the Tins

Enhancement of collectivity for proton-rich tin isotopes

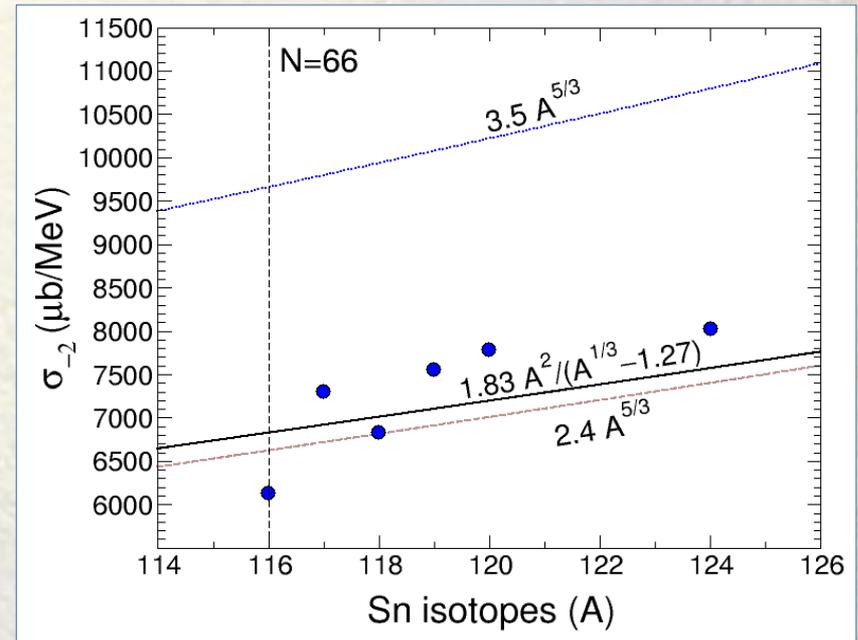
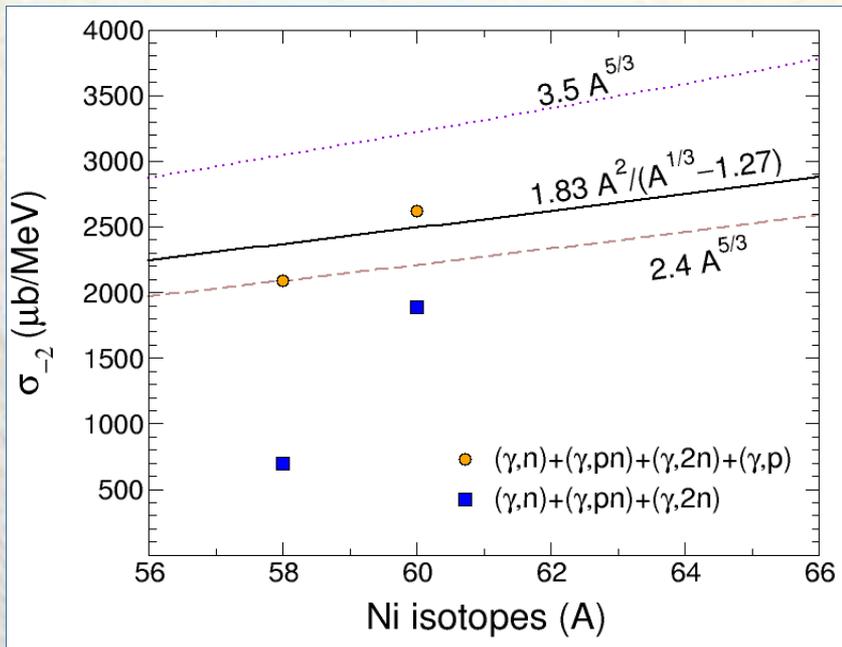


Everything is OK. A small $\kappa < 1$ do not affect/enhance collectivity

J. N. Orce, to be submitted

The remarkable cases of the Nickels and Tins

Discrepancies between lifetimes vs Coulex in ^{58}Ni
and enhancement of collectivity for proton-rich tin isotopes



Everything is OK. A small $\kappa < 1$ do not enhance collectivity

J. N. Orce, to be submitted

CONCLUSIONS & FUTURE WORK

- Dipole polarizability effects strong for light nuclei
 - $\kappa < 1$ do not enhance collectivity
 - Ni and Sn isotopes unaffected
 - The goal is to disentangle RE vs E1 polarizability
 - The only information we know is for ground states.
What happens at higher excitation energies? (Cebó's MSc work)
 - Pulsars & symmetry energy: Synergy between SKA & Coulex
- Lots of new ideas (^{12}C , ^{17}O , ^{18}O , semi-magic nuclei, etc)

VIII Tastes of Nuclear Physics @ UWC

John Wood, Steve Yates, David Jenkins, Paul Garrett, Berta Rubio, Maria Garcia Borge, Alejandro Algora, Magda Zielinska, Dan Doherty, Mark Riley, Paul-Henri Heenen, Carlos Bertulani, Emanuele Vardaci, John Sharpey-Schafer, Mathis Wiedeking, Smarajit Triambak + our UWC Students, etc.



New Physics with Modern African Nuclear Detector Laboratory
(nuclear reactions, new particle-detector arrays, etc)

Motives for Scientific Creativity



ARKADIĬ BENEDIKTOVICH MIGDAL
(1911–1991)

Not for you are passion and goldlust,
It is science that entices you.

Passion may fade and love is betrayed
But you cannot be deceived
By the bewitching structure of the cockroach.

N. Olennikov, *Comic Verses*

On the Psychology of Scientific Creativity

A.B. Migdal, *Contemp. Phys.* VOL. 20, NO. 2, 121-148 (1979)

Estimates of Dipole Polarization Effects

In this appendix we study the dipole polarization in order to estimate the polarization corrections to the elastic and inelastic scattering. The calculations are based on the expressions (II.6.8) and the empirical knowledge of the giant dipole resonance.

We consider a nucleus with ground state spin I_0 and evaluate first the matrix element of V_{pol} between two magnetic substates M_0 and M'_0 . According to eq. (II.6.8) one finds

$$\langle I_0 M_0 | V_{\text{pol}} | I_0 M'_0 \rangle = - \sum_z \frac{\langle I_0 M_0 | V_{\text{E}1}(t) | I_z M_z \rangle \langle I_z M_z | V_{\text{E}1}(t) | I_0 M'_0 \rangle}{E_z - E_0} \quad (1)$$

where the dipole field $V_{\text{E}1}(t)$ is given by

$$V_{\text{E}1}(t) = -\frac{4}{3}\pi |E| \sum_{\mu} (-1)^{\mu} Y_{1-\mu}(E) \mathcal{M}(E1, \mu), \quad (2)$$

and E denotes the electric field strength at the nucleus, i.e.

$$|E| = Z_1 e / r^2. \quad (3)$$

Evaluating eq. (1) one finds

$$\begin{aligned} \langle I_0 M_0 | V_{\text{pol}} | I_0 M'_0 \rangle &= -\frac{4}{3}\pi |E|^2 \sum_z \frac{B(E1, I_0 \rightarrow I_z)}{E_z - E_0} \\ &\times \left\{ \delta_{M_0 M'_0} + (-1)^{I_0 - M'_0} (2I_0 + 1) \begin{pmatrix} I_0 & I_0 & 2 \\ -M_0 & M'_0 & -\kappa \end{pmatrix} \right. \\ &\quad \left. \times \begin{pmatrix} I_0 & I_0 & 2 \\ 1 & 1 & I_z \end{pmatrix} \sqrt{24\pi} Y_{2\kappa}(E) \right\}. \quad (4) \end{aligned}$$

The second term in (4) which depends on the direction of the field E with respect to the nuclear spin vanishes identically for ground state spin $I_0 = 0$

and $I_0 = \frac{1}{2}$. It also vanishes if the dipole mode is weakly coupled to the nuclear spin. In the following we shall neglect this term and write

$$\langle I_0 M_0 | V_{\text{pol}} | I_0 M'_0 \rangle = -\delta_{M_0 M'_0} |E|^2 P, \quad (5)$$

where P is the nuclear polarizability

$$P = \frac{4}{9}\pi \sum_z \frac{B(E1, 0 \rightarrow z)}{E_z - E_0}. \quad (6)$$

This polarizability can be estimated from the minus-two moment σ_{-2} of the photo-nuclear absorption cross section since [LEV 60]

$$P = \frac{\hbar c}{4\pi^2} \sigma_{-2}. \quad (7)$$

Inserting the empirical result

$$\sigma_{-2} = 3.5 A^{5/3} \mu\text{b}/\text{MeV} \quad (8)$$

one finds

$$P = 1.7 \times 10^{-3} A^{5/3} \text{fm}^3. \quad (9)$$

The polarizability may instead be estimated by writing eq. (6) in the form

$$P = \frac{4}{9}\pi \sum_z \frac{B(E1, 0 \rightarrow z)(E_z - E_0)}{(E_z - E_0)^2}. \quad (10)$$

The energy in the denominator may approximately be taken outside the summation and estimated through the known value of the energy of the giant dipole resonance. The rest can then be estimated by the dipole sum rule. Such estimates are in fair agreement with eq. (9).

For a deformed nucleus the giant dipole resonance is known to split, the energy splitting being inversely proportional to the relative change of the nuclear radius of the principal axes. It is seen from eq. (10) that this indicates that also the polarizability P_{ii} in the direction of the i th principal axes is different for $i = 1, 2$ and 3 . In fact, one expects from (10)

$$P_{ii} = P_0 R_i^2 / R_0^2, \quad (11)$$

where P_0 is the polarizability of a spherical nucleus of the radius R_0 , while R_i is the length of the i th axis.

In such a situation the polarizability is a tensor of second rank and the dipole moment which is induced by an external homogeneous electric field is only in the direction of the field if E is parallel to one of the three principal axes.

The situation is indicated in fig. 1 from which it is evident that such a polarizability may give rise to excitation of rotational states.



Fig. 1. A deformed nucleus in an external homogeneous electric field E . The induced dipole moment D is not in the direction of the external field. Its direction depends on the orientation of the principal axes, which are indicated. The torque which is produced by the interaction between E and D may set the nucleus into rotation.

One may estimate the non-diagonal matrix elements of V_{pol} between nuclear states which are described by nuclear quadrupole deformations. We thus assume that the nuclear surface can be described by (see chapter VII)

$$R(\theta, \phi) = R_0 \left[1 + \sum_{\mu} \alpha_{2\mu} Y_{2\mu}^*(\theta, \phi) \right] \quad (12)$$

and consider that the deformation parameters $\alpha_{2\mu}$ only influence the giant dipole resonance in an adiabatic fashion. Since the component of the induced dipole moment D_E in the direction of the external field θ, ϕ is given by

$$D_E = EP_0 R^2(\theta, \phi) / R_0^2. \quad (13)$$

The polarization potential is given by

$$\begin{aligned} V_{\text{pol}} &= -D_E E \\ &= -P_0 E^2 \left[1 + 2 \sum_{\mu} \alpha_{2\mu} Y_{2\mu}^*(E) \right] \end{aligned} \quad (14)$$

to lowest order in $\alpha_{2\mu}$. Assuming the connection (see § VII.1)

$$\mathcal{M}(E2, \mu) = \frac{3}{4\pi} Z_2 e R_0^2 \alpha_{2\mu} \quad (15)$$

between the electric quadrupole moment and the deformation parameter, we may write

$$V_{\text{pol}} = -P_0 \frac{Z_1^2 e^2}{r^4} \left[1 + \frac{8\pi}{3Z_2 e} \sum_{\mu} \frac{1}{R_0^2} \mathcal{M}(E2, \mu) Y_{2\mu}^*(E) \right]. \quad (16)$$

It is expected that eq. (16) gives an estimate of the non-diagonal matrix elements between nuclear states which can be described in terms of collective surface deformation, whether they are of vibrational or rotational nature. The total quadrupole interaction $V_{\text{E2}}(t) + V_{\text{pol}}(t)$ can thus be written as

$$V_{\text{E2}}(t) + V_{\text{pol}}(t) = Z_1 e \frac{4\pi}{5} \sum_{\mu} (-1)^{\mu} \frac{1}{r^3} \mathcal{M}(E2, -\mu) Y_{2\mu}(\theta, \phi) \left[1 - z(1, 1, 2) \frac{a}{r} \right], \quad (17)$$

with

$$z(1, 1, 2) = \frac{10Z_1 P_0}{3Z_2 R_0^2 a} \sim 0.5 \times 10^{-2} \frac{E_{\text{MeV}} A_2}{Z_2^2 (1 + A_1/A_2)}. \quad (18)$$